BT MRE – I&II – 09 – 01

B.Tech Degree I & II Semester Examination in Marine Engineering, May 2009

MRE 101 ENGINEERING MATHEMATICS I

Time : 3 Hours | Maximum Marks : 100

(All questions carry ***EQUAL*** marks)

I. (a) Verify Roll's theorem for the function $f(x) = \frac{Sin\,x}{e^x} in[0,\pi]$. (6)

(b) Evaluate $\lim_{x \to 0}\left(\frac{1}{Sin\,x} - \frac{1}{x}\right)$. (7)

(c) Find the radius of curvature of the cycloid $x = a(\theta + Sin\,\theta); y = a(1 - Cos\theta)$. (7)

OR

II. (a) Find the asymptotes of the curve
$$2x^3 - x^2y - 2xy^2 + y^3 + 2x^2 + xy - y^2x + y + 1 = 0.$$ (8)

(b) If $y = e^{a\,Sin_x^{-1}}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. (12)

III. (a) Given that $u = Sin(x^2 + y^2)$ where $x = 3t$ and $y = \frac{1}{1+t^2}$, determine $\frac{du}{dt}$. (7)

(b) Verify Euler's theorem for the function $u = x^n Sin\frac{y}{x}$. (6)

(c) The power P required to propel a steamer of length l at a speed u is given by $P = \lambda u^3 l^2$, where λ is a constant. If u is increased by 3% and ℓ is decreased by 1%, find the corresponding increase in P. (7)

OR

IV. (a) If $u = x^2 - y^2$ and $v = 2xy$ prove that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{4\sqrt{u^2+v^2}}$. (6)

(b) Prove that $u = x^3 + y^3 - 3axy$ is maximum or minimum at $x = y = a$ according as a is negative or positive. (7)

(c) If $u = Sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7)

V. (a) Find the co-ordinates of the centre, foci and equation to the directrices of the ellipse $9x^2 + 25y^2 - 18x - 100y + 116 = 0$. (6)

(b) Find the equation to the parabola whose focus is the point $(3,4)$ and directrix is the straight line $2x - 3y + 5 = 0$. (6)

(c) The asymptotes of a hyperbola are parallel to $2x+3y=0$ and $3x - 2y = 0$. Its centre is at $(1,2)$ and it passes through the point $(5,3)$. Find its equation. (8)

OR

(Turn Over)

VI. (a) Find the condition that the straight line $Px + my + n = 0$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (7)

(b) Show that the locus of the point of intersection of two perpendicular tangents to a parabola is its direction. (6)

(c) Prove that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact and encloses a triangle of constant area. (7)

VII. (a) Using the double integral, find the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (10)

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$. (10)

OR

VIII. (a) Find the area of the surface of revolution formed by revolving the loop of the curve $9ay^2 = x(3a-x)^2$ about x - axis. (10)

(b) Change the order of integration and their evaluate :

$$I = \int_0^a \int_{\sqrt{(ax)}}^a \frac{y^2 dy\, dx}{\sqrt{(y^4 - a^2x^2)}}.$$ (10)

IX. (a) Show that the vectors $\vec{a} - 2\vec{b} + 3\vec{c}, -2\vec{a} + 3\vec{b} - 4\vec{c}, -\vec{b} + 2\vec{c}$ are coplanar. (6)

(b) Is any triangle ABC, prove that $c^2 = a^2 + b^2 - 2ab\,Cos\,C$. (6)

(c) Given $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$, find $\vec{a} \times \vec{b}$ and a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. Also determine the Sine of the angle between $\vec{a}$ and $\vec{b}$. (8)

OR

X. (a) Prove that $\left[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$. (6)

(b) Find the divergence and eurl of the vector $\vec{V} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at the point $(2,-1,1)$. (6)

(c) Prove that div $\left(\frac{\vec{r}}{r^3}\right) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (8)

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